

## **MATHEMATICAL MODEL FOR OPERATION OF AVIATION SYSTEMS FOR DELIVERY OF SPECIAL MEANS TO AIR AND EARTH OBJECTS**

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### **Abstract**

*The development of technologies allows the improvement of the algorithms for the operation of aviation systems for the delivery of special means to air and ground objects. The aim is to increase their efficiency, while facilitating the work of the crew in case of lack of time to complete the task. A mathematical model for the operation of this type of aviation systems for the supply of special equipment has been developed, which allows reducing the volume of the existing protocol for the implementation of the crew. It is proposed to use a common ballistic algorithm, combining different groups of special means of delivery to air and ground sites.*

### **Introduction**

In modern manned military aircraft, full automation of the entire process from target selection to automatic separation of the weapons is essential [1, 6]. The problem is that the wide variety of weapons and their multivariate use creates additional difficulties for full automation [4, 5]. Solving the ultimate task of bringing the weapons to the target requires taking into account the peculiarities in determining the trajectory of each of the groups of weapons — guided with and without motor, unguided with and without motor. What is more, the type of target — aerial, ground (mobile, stationary), flight characteristics of the aircraft must also be taken into account.

This raises the need to create a common algorithm for the operation of the system for release of all types of weapons, taking into account their characteristics.

Unguided weapons are used at a wide range of aircraft heights and speeds, angles and distances to the target, the presence of an angle between the aircraft speed vector and the initial velocity vector of the weapon. In addition, weapons have fundamentally different ballistic schemes. Specific methods are used to solve ballistic problems and various forms of presenting the results of the solutions [2, 7]. It is necessary to synthesize a common ballistic algorithm for all types of

unguided weapons, when the type of weapon used is selected on board the aircraft by the crew through a weapon control switch.

### Study area

To compile a general mathematical model for the release of unguided weapons (UW) is used the general vector scheme determining the destruction of the target — aerial or ground — by the weapon (Fig. 1).

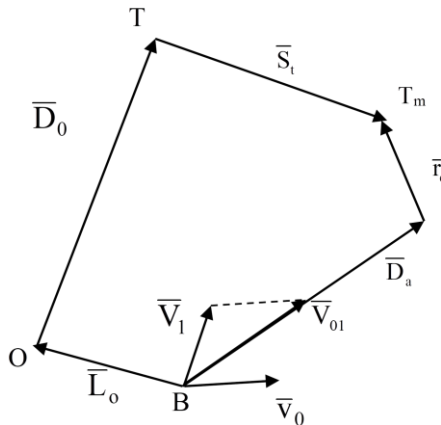


Fig. 1. General vector scheme of aiming

$$1) \quad \vec{D}_0 = \vec{D}_a + \vec{r}_c - \vec{L}_0 - \vec{S}_t,$$

where:

$\vec{L}_0$  is the vector of the base of the weapon;

$\vec{D}_0$  — the target distance vector at the time of the shot;

$\vec{D}_a$  — the vector of the anticipatory distance to the target at the time of the shot;

$\vec{S}_t$  — the vector determining the movement of the target;

$\vec{r}_c$  — the vector determining the deviation of the projectile in the case of a mobile artillery weapon.

The vector equation (1) is represented in scalar form in the connected coordinate system  $Ox_1y_1z_1$ .

The vector of the base  $\vec{L}_0$  of the weapon is determined in the  $Ox_1y_1z_1$  coordinate system:

$$2) \quad \vec{L}_0 = \vec{L}_{0x1} + \vec{L}_{0y1} + \vec{L}_{0z1}.$$

For the synthesis of the general ballistic model, a system of differential equations describing the spatial motion of a weapon in the navigation basis  $O_{\eta\zeta\xi}$  is proposed. The axis  $O_{\eta}$  of the navigation basis is oriented along the meridian to the north, the axis  $O_{\zeta}$  is oriented along the local vertical, and the axis  $O_{\xi}$  is oriented so as to form a right coordinate system [3]:

$$\begin{aligned}
 \frac{dV}{dt} &= \frac{P}{m} - c_k \frac{m_k}{m d^2} H(y) S V^2 C_{xe}(M) 10^{-3} - g \sin \lambda_b; \\
 \frac{d\lambda_b}{dt} &= -g \sin \lambda_b \frac{1}{V} + C_y^\alpha(M) (\alpha - \alpha_0) H(y) \frac{\rho_0 V}{2m} S; \\
 \frac{d\eta}{dt} &= V \cos \lambda_b; \\
 \frac{d\xi}{dt} &= V \sin \lambda_b,
 \end{aligned}$$

where:

$P$  is the thrust of the rocket's engine;

$V$  — the speed of the weapon;

$m_k$  — the mass of the rocket after the engine shut down;

$c_k$  — the ballistic coefficient of the rocket in the passive section.

The vector of the anticipatory distance is presented in the connected coordinate system  $Ox_1y_1z_1$  after solving the ballistic problem with the general model of motion — form. (2):

$$(\bar{D}_a)_1 = \left[ A_{\eta\zeta\xi}^{(x_1y_1z_1)} \right]^T \begin{bmatrix} \eta \\ \zeta \\ \xi \end{bmatrix};$$

When firing from a mobile artillery installation, the projectile leaves the barrel with a nutation angle  $\delta_0$ . In the general case, the initial velocity  $\vec{v}_0$  vector of the projectile (relative to the aircraft) and the velocity vector  $\vec{V}_1$  do not match.

The deviation of the projectile  $r_c$  is determined [3, 5] by the formula:

$$(\bar{r}_c)_1 = C_r \begin{bmatrix} \cos \beta_s \sin \alpha \sin \beta' \cos \varepsilon' - \sin \beta_s \sin \varepsilon' \\ \sin \beta_s \cos \beta' \cos \varepsilon' + \cos \beta_s \cos \alpha \sin \beta' \cos \varepsilon' \\ \cos \beta_s \cos \alpha \sin \beta' \sin \varepsilon' + \cos \beta_s \sin \alpha \cos \beta' \cos \varepsilon' \end{bmatrix},$$

where  $C_r = \frac{c_y^\delta}{m_{z1}^\delta} \frac{d}{l} \frac{\pi \mu}{2 \eta_b V_{01}^2} D$ .

In the case of a moving target, the following hypothesis for the movement of the target is accepted:

$$\dot{\vec{V}}_t = const.$$

The projection of the vector  $\vec{S}_t$  on the axes of the connected system  $Ox_1y_1z_1$  is determined by the expression:

$$7) \quad \vec{S}_t = \left\{ \left[ A_v^{(1)} \right]^T \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} + A_1^{(xDyDzD)} \begin{bmatrix} \dot{D} \\ \omega_{zD}D \\ -\omega_{yD}D \end{bmatrix} \right\} T + \frac{1}{2} \begin{bmatrix} a_{vx1} \\ a_{vy1} \\ a_{vz1} \end{bmatrix} T^2 + \\ + \frac{1}{2} A_1^{(xDyDzD)} \begin{bmatrix} \ddot{D} - D(\omega_{yD}^2 + \omega_{zD}^2) \\ \omega_{xD}\omega_{yD}D + \dot{\omega}_{zD}D + 2\omega_{zD}\dot{D} \\ \omega_{xD}\omega_{zD}D - \dot{\omega}_{yD}D - 2\omega_{yD}\dot{D} \end{bmatrix} T^2.$$

Putting in form (1) forms (2), (4), (5) and (7) the general system of scalar equations for calculating the distance to the target at the time of the shot is obtained:

$$8) \quad \begin{bmatrix} D_{0x1} \\ D_{0y1} \\ D_{0z1} \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + C_r \begin{bmatrix} \cos \beta_s \sin \alpha \sin \beta' \cos \varepsilon' - \sin \beta_s \sin \varepsilon' \\ \sin \beta_s \cos \beta' \cos \varepsilon' + \cos \beta_s \cos \alpha \sin \beta' \cos \varepsilon' \\ \cos \beta_s \cos \alpha \sin \beta' \sin \varepsilon' + \cos \beta_s \sin \alpha \cos \beta' \cos \varepsilon' \end{bmatrix} - \\ - \begin{bmatrix} L_{0x1} \\ L_{0y1} \\ L_{0z1} \end{bmatrix} - \left\{ \left[ A_v^{(1)} \right]^T \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} + A_1^{(xDyDzD)} \begin{bmatrix} \dot{D} \\ \omega_{zD}D \\ -\omega_{yD}D \end{bmatrix} \right\} T - \\ - \frac{1}{2} \begin{bmatrix} a_{vx1} \\ a_{vy1} \\ a_{vz1} \end{bmatrix} T^2 + \frac{1}{2} A_1^{(xDyDzD)} \begin{bmatrix} \ddot{D} - D(\omega_{yD}^2 + \omega_{zD}^2) \\ \omega_{xD}\omega_{yD}D + \dot{\omega}_{zD}D + 2\omega_{zD}\dot{D} \\ \omega_{xD}\omega_{zD}D - \dot{\omega}_{yD}D - 2\omega_{yD}\dot{D} \end{bmatrix} T^2.$$

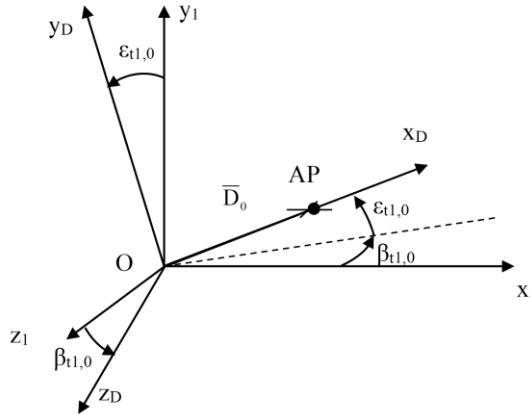


Fig. 2. Aiming point (AP) and aiming angles of the target  $\beta_{t1,0}$ ,  $\varepsilon_{t1,0}$  at the moment of firing and bombing

In the general case at the moment of firing and bombing, the angles  $\beta_{t1,0}$  and  $\varepsilon_{t1,0}$  of aiming at the target in the connected coordinate system  $Ox_1y_1z_1$  (Fig. 2) are determined by the formulas:

$$9) \quad \beta_{t1,0} = \arctg\left(\frac{D_{0z1}}{D_{0x1}}\right) \quad \varepsilon_{t1,0} = \arctg\left(\frac{D_{0y1} \sin \beta_{t1,0}}{D_{0z1}}\right),$$

where:

$$10) \quad D_0 = \sqrt{D_{0x1}^2 + D_{0y1}^2 + D_{0z1}^2}.$$

The firing of aviation automatic weapons and unguided rockets is performed by the pilot combining the calculated aiming angles of the target  $\beta_{t1,0}$  and  $\varepsilon_{t1,0}$  with the visually detectable target while observing the allowed firing distance. This distance is determined by the type of the weapon used.

Depending on the combat capabilities of the aircraft, its design features and the perfection of its strike complex, bombing can take place in the visible zone and in the invisible zone.

When bombing takes place in the visible zone, i.e. The Continuously Computed Impact Point (CCIP) method is used, similarly to shooting, the pilot combines the calculated angles  $\beta_{t1,0}$  and  $\varepsilon_{t1,0}$  in form (9) with the target and depresses the combat button in compliance with safety measures [2, 6].

When bombing into the invisible zone using the Continuously Computed Release Point (CCRP) method, the pilot aligns the aiming grid with the target, as a result of which the calculation of the current coordinates of the target relative to the aircraft begins:

$$11) \quad \begin{cases} \eta_c = \eta_0 - \int_0^t W_\eta dt; \\ \xi_c = \xi_0 - \int_0^t W_\xi dt, \end{cases}$$

where  $W_\eta$ ,  $W_\xi$  are the projections of the ground speed of the aircraft, and  $\eta_0$ ,  $\xi_0$  are the initial coordinates of the target.

The bombing is performed in automatic mode when the inequality is fulfilled:

$$12) \quad \eta_c - \eta \leq 0.$$

Based on the abovementioned general mathematical model for combat use of weapons, a general structural scheme of a weapons release system is proposed (Fig. 3).

The general block diagram of a weapons release system consists of:  
- targeting unit;

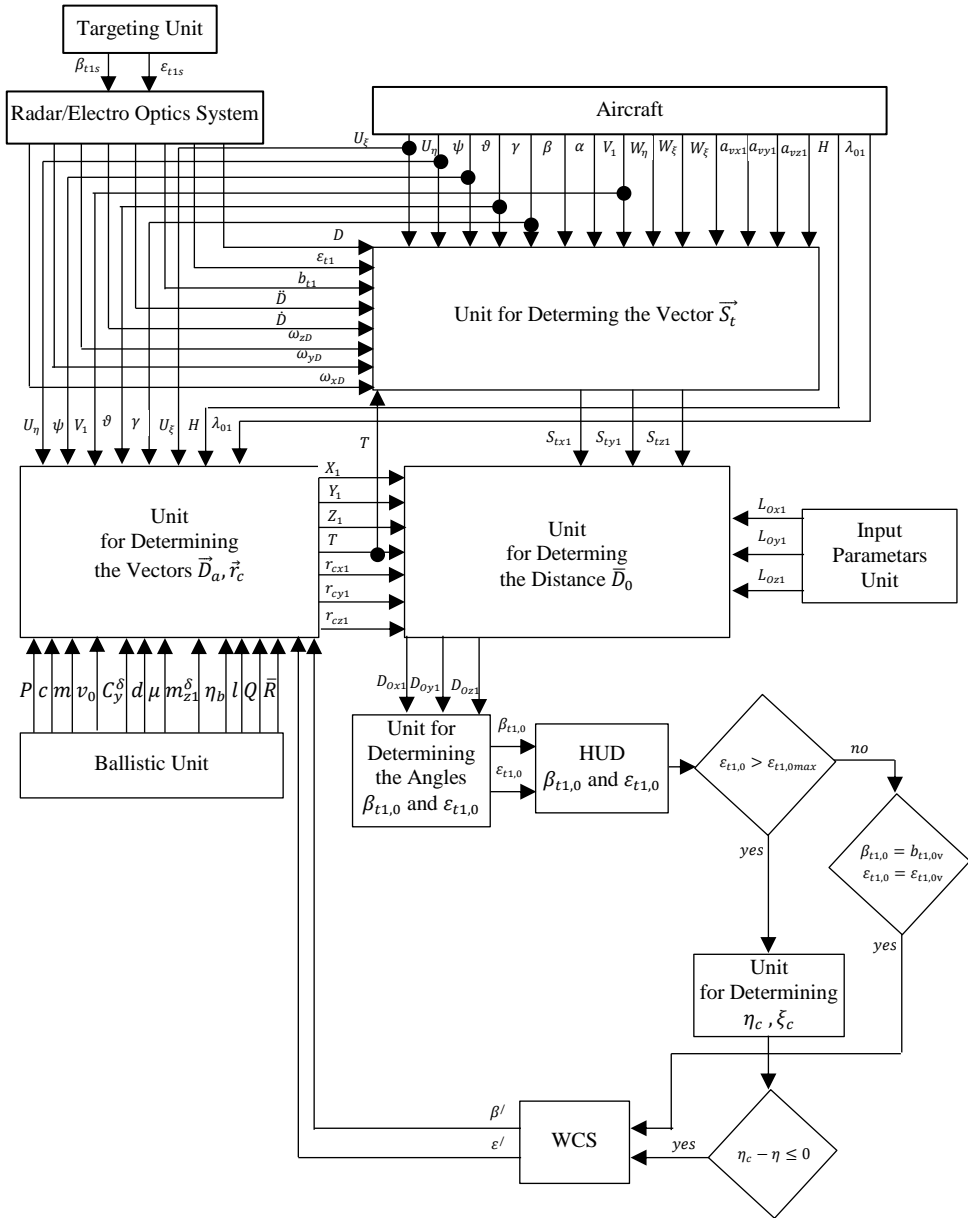


Fig. 3. General structural scheme of a weapons release system

- a unit for determining the vector which determines the movement of the target  $\vec{S}_t$ ;

- a unit for determining the vectors of the anticipatory distance  $\overline{D}_a$  to the target and the deviation  $\overline{r}_c$  of the weapon;
- unit for determining the distance  $\overline{D}_0$  to the target at the time of release;
- ballistic unit;
- input parameters unit;
- unit for determining the angles  $\beta_{t1,0}$  и  $\varepsilon_{t1,0}$ ;
- unit for determining  $\eta_c$ ,  $\xi_c$  of the current coordinates of the target relative to the aircraft;
- Heads-up Display (HUD).

The azimuth angle  $\beta_{\Pi 1}$  and the angle  $\varepsilon_{\Pi 1}$  of the target location are fed from the targeting unit to the R/OES.

The vector determining the movement of the target  $\overline{S}_t$  is calculated after solving the ballistic problem using the system (4) and formula (7) depending on whether the target is aerial or ground.

In the unit for determining the vectors  $\overline{D}_a$  and  $\overline{r}_c$ , they are determined by formulas (4) and (5). Provided that the aviation artillery weapon is stationary, the vector  $\overline{r}_c$  is not calculated.

With the help of the obtained formula (8) the distance to the target  $\overline{D}_0$  at the moment of release is determined in the respective unit.

The values  $P$ ,  $c$ ,  $m$ ,  $v_0$ ,  $C\delta y$ ,  $d$ ,  $\mu$ ,  $m\delta z_1$ ,  $\eta_b$ ,  $l$ ,  $\theta$  and the coordinates of the point of explosion -  $R\eta$ ,  $R\zeta$ ,  $R\xi$  are supplied by the ballistic unit depending on the weapon used.

The base of the aviation artillery weapon  $(\overline{L}_0)_1$  is supplied by the input parameters unit.

The angles of rotation  $\beta'$  and  $\varepsilon'$  relative to the  $Ox_1y_1z_1$  system are supplied by the weapon control system (WCS).

The target aiming angles  $\beta_{t1,0}$  and  $\varepsilon_{t1,0}$  are calculated using form (9) in the relevant unit.

When firing and bombing in the visible zone (CCIP), the calculated angles  $\beta_{t1,0}$  and  $\varepsilon_{t1,0}$  are displayed on the HUD. This is the position of the aiming grid. When the aiming grid is aligned with the target, the pilot depresses the combat button, which signals the weapon control system (WCS). In the case of bombing in the invisible zone (CCRP), after aligning the aiming grid with the target, the current coordinates  $\eta_t$ ,  $\xi_t$  of the target relative to the aircraft are calculated in the respective unit. Upon fulfilment of the condition  $\eta_t - \eta \leq 0$ , a signal is sent to the WCS [5].

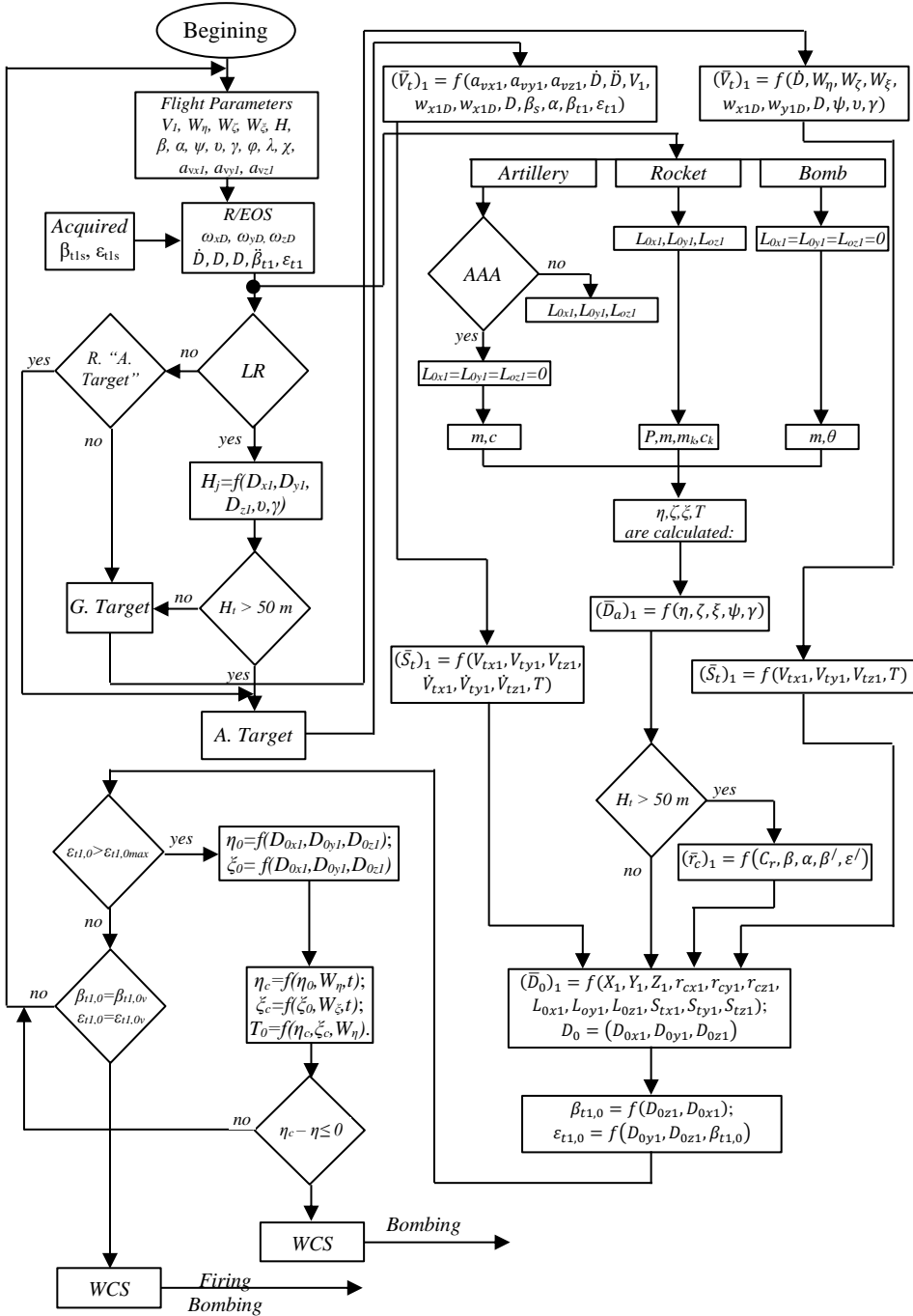


Fig. 4. Operatoin algorithm for combat when shooting and bombing with unguided weapons



Based on the developed general mathematical model and general structural scheme of the weapons release system, an algorithm for combat use has been developed (Fig. 4).

After the target is acquired by the radar or the OES, it is automatically determined what the target is — aerial or ground. In case the acquisition is performed by OES, with the help of the laser rangefinder (LR), the distance  $D$  to the target is measured and its height  $H_t$  is determined using the formula:

$$13) \quad H_t = H_a - D_{x1} \sin \vartheta - D_{y1} \cos \vartheta \cos \gamma + D_{z1} \cos \vartheta \sin \gamma.$$

Provided that  $H_t > 50$  m, a decision is made – it is an aerial target. If the condition is not met – it is a ground target.

When using a radar, the pilot manually selects the “aerial target” or “ground target” mode.

Depending on the type of target, the speed  $V_t$  of the target is calculated.

Depending on the target, the pilot selects the type of weapon. Then the respective parameters included in the system of differential equations for the movement of the weapons are submitted. After solving the system, the vector  $\overline{D}_t$  of the anticipatory distance to the target at the moment of the shot and the time  $T$  of the flight of the weapon are calculated.

In the case of bombing and firing of aviation artillery weapons located along the axis of the aircraft (AAA), it is assumed that the vector of the base on the weapon is:  $\overline{L}_0 \neq 0$ .

When firing a rocket and aviation artillery weapon located in a container attached to the aircraft, it is assumed that  $\overline{L}_0 \neq 0$ .

Depending on the type of target (aerial or ground), the vector  $\overline{S}_t$  determining the movement of the target is calculated. Provided the target is stationary  $\overline{S}_t = 0$ .

Provided that the weapon is mobile ( $\beta' \neq 0$ ;  $\varepsilon' \neq 0$ ) the vector determining the deviation  $\overline{r}_c$  of the projectile is determined.

The next step is to calculate the distance vector  $\overline{D}_0$  to the target at the time of the shot, from which the angles  $\beta_{t1,0}$  and  $\varepsilon_{t1,0}$  of aiming at the target relative to the connected coordinate system  $Ox_1y_1z_1$  are determined.

Provided that shooting or bombing is carried out in the visible zone ( $\varepsilon_{t1,0} < \varepsilon_{t1,0max}$ ) and after aligning the aiming grid with the target ( $\beta_{t1,0} = \beta_{t1,0v}$ ;  $\varepsilon_{t1,0} = \varepsilon_{t1,0v}$ ) a signal is sent to the weapon control system, followed by firing or bombing.

Provided that the bombing is carried out in the invisible zone ( $\varepsilon_{t1,0} > \varepsilon_{t1,0max}$ ), the initial  $\eta_0$ ,  $\xi_0$  and the current coordinates  $\eta_t$ ,  $\xi_t$  of the target relative to the aircraft are determined.

When the condition  $\eta_t - \eta \leq 0$  is met, the bombing is performed automatically.

Provided that  $\eta_t - \eta > 0$  or  $\beta_{t1,0} \neq \beta_{t1,0v}$ ;  $\varepsilon_{t1,0} \neq \varepsilon_{t1,0v}$  then the process continues and returns to the starting position.

## Conclusion

The creation of a common model of a weapons release system, including a common ballistic model of weapons will lead to:

- shortening the operating time of the pilot in case of combat use;
- increasing the efficiency of the use of weapons by expanding the range of conditions of combat use.

## References

1. Mitev, S. Accuracy of Navigation Complexes, Scientific Research and Education in the Air Force — AFASES 2018, Publishing House of “Henri Coanda” Air Force Academy Str. Mihai Viteazu 160, Brasov 500183, Romania — 2018, ISSN: 2247-3173, DOI:10.19062/2247-3173.2018.20.
2. Petrov, N., Sv. Asenov, D. Guinchev, N. Zagorski, Ang. Chzhgova. Assessment of the rate of increase of the legally defined technical resources to the aviation system. *International Journal of Mathematic Sciences and Engineering Applications (IJIMSEA)*, Vol. 7, No VI 9 (November 2013), ISSN 0973-9424, pp. 1–6.
3. Stojkov, O., M. Atanasov. *Aviacionni pricelni sistemi — Part 1*. VGC, D. Mitropolija, 2009, ISBN 978-954-713-093-7 (in Bulgarian).
4. Stojkov, S. Analiz na prichinite vliyaeshti na efektivnostta na boñnoto izpolzvanie na aviatsionnite bombi, NVU “V. Levski”, fakultet “Aviacionen”, D. Mitropolija, “Godishna mezhduнародna nauchna konferentsiya”, pp. 300–303, 2019, ISBN 978-954-713-123-1 (in Bulgarian).
5. Zagorski, N., Ang. Chozhgova, Research on neural network for identification of the status of complex technical objects. *Anniversary Scientific Conference on Aeronautics, Automotive and Railway Engineering and Technologies BulTrans-2013*, 16–18 October, Sofia, Proceedings, pp. 161–164.
6. Zagorski, N. Problemi na nadejdnostta na aviatsionni sistemi i nejnoto vlianie varhu bezopasnostta na poleta, Monografia, IK “Rakursi” - Plovdiv, 2021, ISBN 978-619-7599-03-9, 213 p. (in Bulgarian).
7. Zagorski, N. Priblijeni modeli za optimizatsia na parametrite za eksploatsia i technicheskо obslujvane na aviatsionnite sistemi s otsenka na tochnostta i dostovernostta na poluchavanite rezultati, *Journal of the Technical University at Plovdiv “Fundamental Sciences and Applications”*, Vol. 14(2), 2009, pp. 359–364 (in Bulgarian).

# МАТЕМАТИЧЕСКИ МОДЕЛ ЗА РАБОТА НА АВИАЦИОННИ СИСТЕМИ ЗА ДОСТАВКА НА СПЕЦИАЛНИ СРЕДСТВА ДО ВЪЗДУШНИ И ЗЕМНИ ОБЕКТИ

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## Резюме

Развитието на технологиите позволява усъвършенстване на алгоритмите за работа на авиационните системи за доставка на специални средства до въздушни и земни обекти. Целта е да се повиши тяхната ефективност, като същевременно се улесни работата на екипажа при недостиг на време за изпълнение на задачата. Разработен е математически модел за работа на този тип авиационни системи за доставка на специални средства, който позволява намаляване на обема на съществуващия протокол за изпълнение от екипажа. Предложен е за използване общ балистичен алгоритъм, обединяващ различните групи специални средства за доставка до въздушни и наземни обекти.